Calculus AB

5-2

ln(x): Integration

Integration of $f(x) = \frac{1}{x}$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

Find the indefinite integral. (pg 340)

2) $\int \frac{10}{x}\,dx = 10\ln|x| + C$

$\boxed{\ln x^{10} + C}$

Absolute Value not needed for x^{10} cannot be negative.

*) $\frac{1}{2}\int \frac{2x}{x^2+1}\,dx$ $\quad u = x^2+1$, $du = 2x\,dx$

$\frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|x^2+1| + C$

$\boxed{= \ln\sqrt{x^2+1} + C}$

Using the laws of logarithms, we can bring the half from the front up to the power. The half power becomes the radical.

Absolute Value not needed because the radical cannot have a negative answer.

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16) $\int \frac{2x^2 + 7x - 3}{x - 2}\,dx$

Since we do not have a quotient rule for integrals, we need to divide the equation down. I used synthetic division below to rewrite the integrand (stuff in integral).

$$\begin{array}{r|rrr} & 2x^2 & 7x & -3 \\ & & 4 & 22 \\ \hline 2 & 2x & 11 & 19 \end{array}$$

$= \int \left(2x + 11 + \frac{19}{x-2}\right)dx$

The remainder is always written as a fraction with the divisor.

$\boxed{x^2 + 11x + 19\ln|x-2| + C}$

24) $3\int \frac{1}{3}\cdot\frac{1}{x^{\frac{2}{3}}(1 + x^{\frac{1}{3}})}\,dx$ $\quad u = 1 + x^{\frac{1}{3}}$, $du = \frac{1}{3}x^{-\frac{2}{3}}\,dx$

$3\int \frac{1}{u}\,du$

$3\ln|1 + x^{\frac{1}{3}}| + C$

$\boxed{\ln\left|\sqrt[3]{1+\sqrt[3]{x}}\right| + C}$

Since cube roots may have negative solutions, we must keep the absolute value in the natural logarithm.

Assignment:
Day 1

pg 330
2-29 odd

Integration of Trigonometric Functions

$\int \sin x\,dx =$ $\qquad\qquad$ $\int \cos x\,dx =$

$\int \tan x\,dx = -\int \frac{-\sin x}{\cos x}\,dx$ $\quad u = \cos x$, $du = -\sin x\,dx$

$= -\int \frac{1}{u}\,du = -\ln|\cos x| + C$ or $\ln|\cos x|^{-1} + C = \ln|\sec x| + C$

$-\ln|\cos x| + C$ ← Book uses this one. I use this one.

$\int \cot x\,dx = \int \frac{\cos x}{\sin x}\,dx$ $\quad u = \sin x$, $du = \cos x\,dx$ $\quad \int \frac{1}{u}\,du$

$\boxed{= \ln|\sin x| + C}$

$\int \sec x\, dx =$

$\ln|\sec x + \tan x| + C$

$\int \csc x\, dx =$

$-\ln|\csc x + \cot x|$

The proofs for these theorems are in the text.
I skipped them in class to save time in the lesson.
They are quite interesting, and I recommend reading them!

Find the indefinite integral.

34) $2\int \frac{1}{2}\sec(\frac{x}{2})\, dx =$ $\quad u = \frac{x}{2}$, $du = \frac{1}{2}dx$

$2\int \sec u\, du$

$\boxed{2\ln\left|\sec\frac{x}{2} + \tan\frac{x}{2}\right| + C}$

$\boxed{\ln\left(\sec\frac{x}{2} + \tan\frac{x}{2}\right)^2 + C}$

Solve the differential equation.

44) $\frac{dy}{dx} = \frac{2x}{x^2 - 9}$; (0, 4)

$\int dy = \int \frac{2x}{x^2-9}\, dx$ $\quad u = x^2 - 9$, $du = 2x\,dx$

$y = \ln|x^2 - 9| + c$

$4 = \ln|0 - 9| + c$

$4 = \ln 9 + c$

$4 - \ln 9 = c$

$y = \ln|x^2 - 9| + 4 - \ln 9$ or $\boxed{\ln\frac{|x^2-9|}{9} + 4}$

Evaluate the definite integral. Check using the graphing calculator.

54) $\frac{1}{2}\int_{-1}^{1} \frac{2 \cdot 1}{2x+3}\, dx = \frac{1}{2}\int_{1}^{5} \frac{1}{u}\, du = \frac{1}{2}\ln|u| \Big|_{1}^{5}$

$= \frac{1}{2}(\ln 5 - \cancel{\ln 1})$ $\quad 0$

$= \ln\sqrt{5}$

Using the laws of logarithms,
we can bring the half from the front
up to the power. The half power
becomes the radical.

Alternate solution substituting
the x back in for the u, keeping
the original limits.

$\frac{1}{2}\ln|2x+3| \Big|_{-1}^{1}$

$\frac{1}{2}(\ln 5 - \ln 1)$

$\ln\sqrt{5}$ $\quad$ ln(1) = 0

Find $f'(x)$.

68) $f(x) = \int_0^x \tan t\, dt$

Use the second fundamental theorem
of calculus...

$F'(x) = \tan x$

Find the average value of the function over the interval.

*) $f(x) = \frac{\ln x}{x}$; $[1, e]$

$\frac{1}{e-1}\int_1^e \frac{\ln x}{x}\, dx$ $\quad u = \ln x$, $du = \frac{1}{x}dx$

$\frac{1}{e-1}\int_0^1 u\, du$ $\quad \frac{1}{e-1}\left[\frac{1}{2}u^2\right]_0^1$

$\frac{1}{2(e-1)}[1 - 0]$

$\boxed{\frac{1}{2e-2}}$

Assignment:
Day 2

pg 330

31-59 odd
67, 69,
73-83 odd.